

Narrowing of saturation resonance in collisional plasma

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ABSTRACT

The profile of nonlinear resonance in the probe-field ionic spectrum is shown to narrow down with the saturating field detuning. Physical cause of the effect is the falling velocity dependence of Coulomb collision frequency.

1 INTRODUCTION

Probe-field technique allows one to recover the scattering parameters in the gas or plasma medium measuring width and shift of dips and peaks in the distribution of populations. The velocity dependence of the collision frequency can lead to a significant manifestation when the detuning of pumping field is nearly Doppler width, since the light interacts with fast particles. In plasma the scattering results in the Fokker-Plank diffusion in the velocity space. The profile of nonlinear resonance in ionic spectra was evaluated¹ assuming the constant collision frequency. One must estimate the velocity dependence effects and tensor structure of the diffusion as a next step. The Chandrasekhar velocity dependence of the diffusion tensor for equilibrium plasma² considerably modifies³ the value of the Dicke narrowing⁴ of spectral line. The main objective of present paper is to study the nonlinear case in the pumping field intensity.

2 BASIC EQUATIONS

We consider the situation when both saturating and probe field interact with the same transition between two ionic levels. The equation for density matrix ρ_{ij} of the two-level system can be written as

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \nabla + \Gamma_{ij} \right) \rho_{ij} = q_j(\mathbf{v}) \delta_{ij} + \nu \hat{B} \rho_{ij} - i [\hat{V}, \rho]_{ij},$$

where $i, j = m, n$; $\Gamma_{jj} \equiv \Gamma_j$, $\Gamma_{mn} \equiv \Gamma$ are the relaxation constants of j -level population and coherence, correspondingly; $q_j(\mathbf{v})$ is the excitation function of level j ; \hat{V} is the hamiltonian operator of interaction of the two-level system and the field;

$$\nu = \frac{16\sqrt{\pi} N_i e^4 L}{3m^2 v_T^3}$$

is the effective transport collision frequency, L is the Coulomb logarithm; e, m are the ion charge and mass, N_i, v_T are ionic density and thermal velocity,

$$\hat{B} = \frac{1}{2} \frac{\partial}{\partial \xi_\alpha} \Phi_{\alpha\beta}(\boldsymbol{\xi}) \left(\frac{\partial}{\partial \xi_\beta} + 2\xi_\beta \right)$$

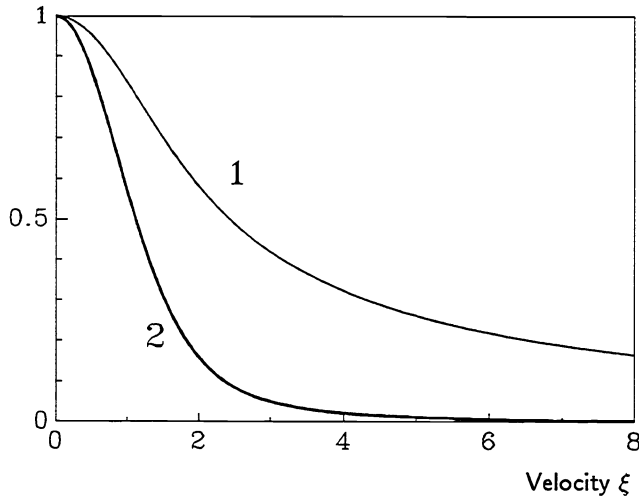


Fig.1. The dependence of transverse (curve 1) and longitudinal (curve 2) diffusion coefficients on the velocity value. Both the dependencies are decreasing, the longitudinal coefficient falls faster.

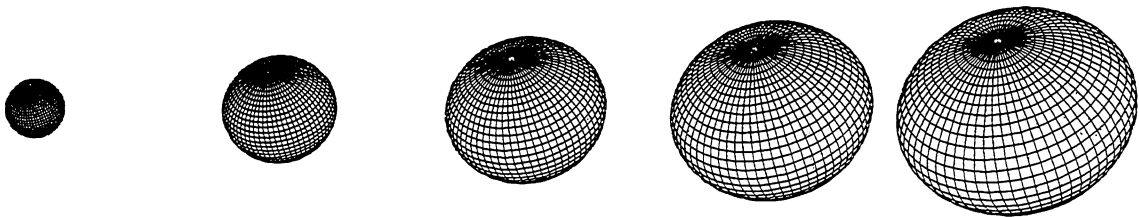


Fig.2. Level surfaces of the function $v_\alpha v_\beta \Phi_{\alpha\beta}^{-1}(\mathbf{v})$, corresponding to the values $0.5^2, 1^2, 1.5^2, 2^2$ and 2.5^2 . If the collision frequency is constant then this surfaces are spherical. The different velocity dependencies of longitudinal and transverse coefficients make the surfaces oblate.

is the collisional operator; $\xi = \mathbf{v}/v_T$ is the velocity normalized by the thermal velocity,

$$\Phi_{\alpha\beta}(\xi) = \Phi_{\parallel}(\xi) \frac{\xi_\alpha \xi_\beta}{\xi^2} + \Phi_{\perp}(\xi) \left(\delta_{\alpha\beta} - \frac{\xi_\alpha \xi_\beta}{\xi^2} \right)$$

is the diffusion tensor. Its longitudinal and transverse components depending on the velocity absolute value only can be written as single integrals over parameter λ (see Fig.1, 2)

$$\Phi_{\parallel}(\xi) = 3 \int_0^1 d\lambda \lambda^2 e^{-\lambda^2 \xi^2}, \quad \Phi_{\perp}(\xi) = \frac{3}{2} \int_0^1 d\lambda (1 - \lambda^2) e^{-\lambda^2 \xi^2}.$$

3 PROFILE OF NONLINEAR RESONANCE

This problem could be solved by the perturbation theory in pumping field owing to a simple commutation relation $[\hat{A}, [\hat{A}, [\hat{A}, \hat{B}]]] = 0$ between relaxation $\hat{A} = \Gamma - i\Omega + i\mathbf{k}\mathbf{v}$ and collisional operator. Here Ω is the

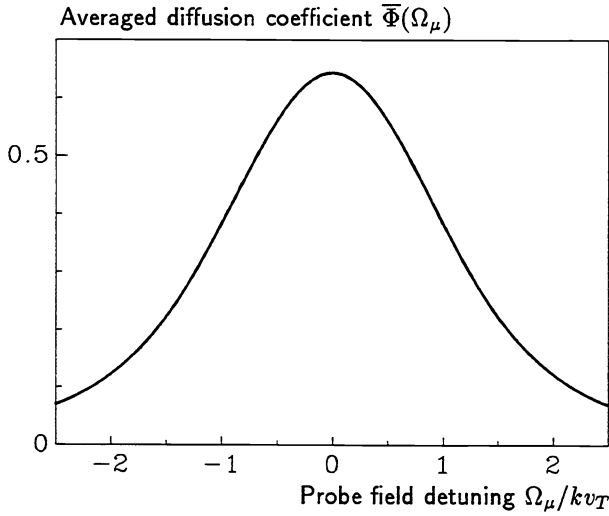


Fig.3. The diffusion coefficient $\Phi_{\mathbf{k}\mathbf{k}}$ averaged over transverse velocities as a function of the probe field detuning Ω_μ . Its asymptotic is $\bar{\Phi} \propto \Omega_\mu^{-3}$, $\Omega_\mu \gg kv_T$.

detuning between saturating field and the transition, \mathbf{k} is the wavevector of pumping field. The relation is specific to the diffusion nature of the scattering.

Under assumption $\Gamma_j (\Gamma/kv_T)^2 \ll \nu \ll \Gamma_j \ll kv_T$ typical to gas discharge plasma we obtain the following expression for addition to the probe field absorption spectrum for counterpropagating waves

$$\mathcal{P}_\mu^{\uparrow\downarrow}(\Omega_\mu) \propto e^{-\frac{\Omega_\mu^2}{(kv_T)^2}} \sum_{j=m,n} \operatorname{Re} \int_0^\infty dt \frac{\exp\left(-2\Gamma t - i(\Omega + \Omega_\mu)t - \frac{\nu(kv_T)^2}{3} \bar{\Phi}(\Omega_\mu)t^3\right)}{\Gamma_j + \frac{\nu(kv_T)}{2} \bar{\Phi}(\Omega_\mu)t^2},$$

where Ω_μ is the probe field detuning,

$$\bar{\Phi}(\Omega_\mu) = \langle \Phi_{\mathbf{k}\mathbf{k}}(\xi) \rangle |_{\mathbf{k}\mathbf{v}=\Omega_\mu},$$

$$\Phi_{\mathbf{k}\mathbf{k}}(\xi) = \Phi_{\parallel}(\xi) \frac{(\mathbf{k}\mathbf{v})^2}{k^2 v^2} + \Phi_{\perp}(\xi) \frac{[\mathbf{k} \times \mathbf{v}]^2}{k^2 v^2}$$

is the projection of the diffusion tensor onto the wavevector, averaged over transverse velocity and taken at resonant velocity.

While the Coulomb broadening greater than Γ and less than Doppler width kv_T , the root-mean-square width

$$\Delta\Omega_\mu = kv_T \sqrt{\frac{\nu \bar{\Phi}(\Omega)}{\Gamma_j}}$$

depends also on the projection of the tensor onto the wavevector \mathbf{k} . With increase in detuning Ω the field interacts with faster ions for which the diffusion tensor is less. Thus the nonlinear resonance becomes narrower as well as the Bennett hole.⁵ In accordance with our qualitative consideration the narrowing depends on homogeneous line width. The effect is strongest at zero Γ (see Fig.4). For ion-laser plasma experimental conditions at the detuning nearly the Doppler width the narrowing factor expected is about 10% (Fig.5).

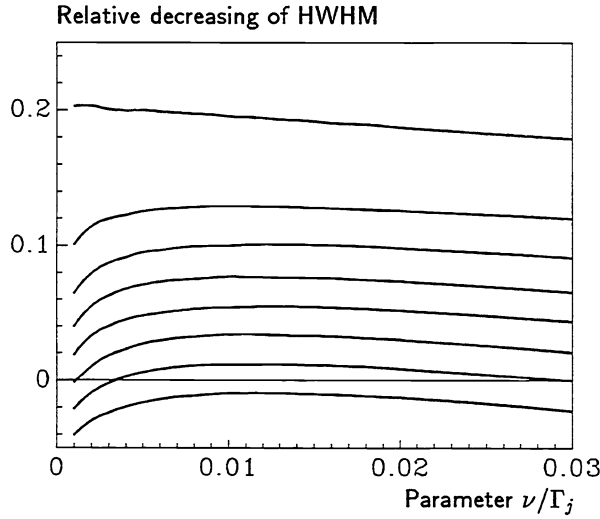


Fig.4. Relative decreasing of HWHM for nonlinear resonance in the probe-field spectrum as a function of parameter ν/Γ_j while detuning Ω increases from 0 to kv_T . Different curves correspond to different values of homogeneous width Γ . This width varies from top to bottom from 0 to $7 \times 10^{-2} kv_T$ with step $10^{-2} kv_T$.

For corresponding waves the additional resonance caused by nonlinear interference appears $\mathcal{P}_\mu^{\uparrow\downarrow}(\Omega_\mu) = \mathcal{P}_\mu^{\uparrow\downarrow}(-\Omega_\mu) + \delta\mathcal{P}^{\uparrow\downarrow}$

$$\delta\mathcal{P}^{\uparrow\downarrow} \propto e^{-\frac{\Omega_\mu^2}{(kv_T)^2}} \sum_{j=m,n} \operatorname{Re} \int_0^\infty dt \frac{\exp\left(-2\Gamma t - i(\Omega - \Omega_\mu)t - \frac{\nu(kv_T)^2}{3} \overline{\Phi}(\Omega_\mu)t^3\right)}{\Gamma_j + i(\Omega - \Omega_\mu) + \frac{\nu(kv_T)}{2} \overline{\Phi}(\Omega_\mu)t^2},$$

The shape of additional resonance from narrow level $\Gamma_j \ll \Gamma$ is a narrow peak or dip at $\Omega_\mu = \Omega$ changing its sign when Ω_μ deviates from Ω (**Fig.6**).

4 DISCUSSION

A diversity of experiments was carried out in argon low-temperature plasma under conditions of considerable Coulomb broadening. The Lamb dip profile in visible and ultraviolet regions, probe field spectrum and spontaneous emission with the strong field presented on adjacent transition, generation of the plasma-based Raman laser were measured (see references in paper⁶). In each case it is difficult to select the Coulomb effect from the total collisional broadening. To clear up the role of charged particles requires independent diagnostics of electron concentration. An advantage of the narrowing effect analyzed in present paper consists in its specifics to Coulomb scattering. The effect makes it possible elucidating the Coulomb relative contribution into the whole cross-section and to determine the degree of ionization. A direct measurement of the component of Chandrasekhar's diffusion tensor in the velocity space is in sight.

5 ACKNOWLEDGMENTS

The research described in this publication was possible in part by Grant No. RCN000 from the International Science Foundation.

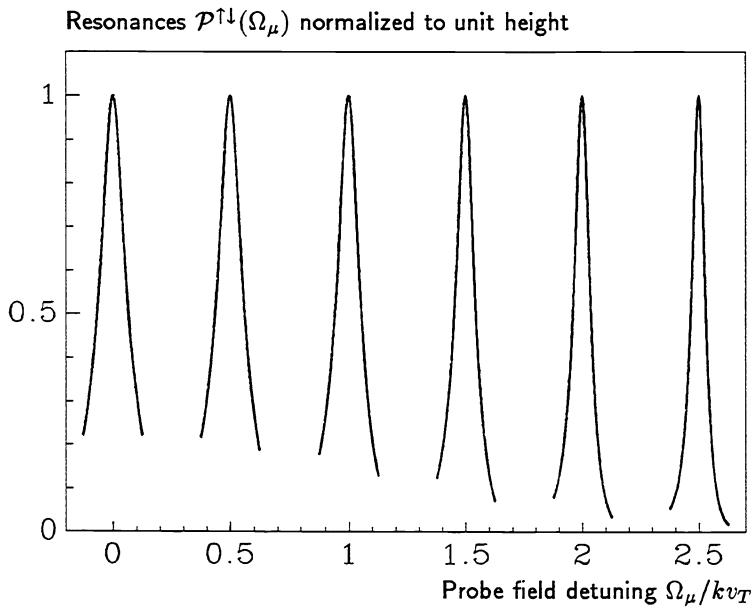
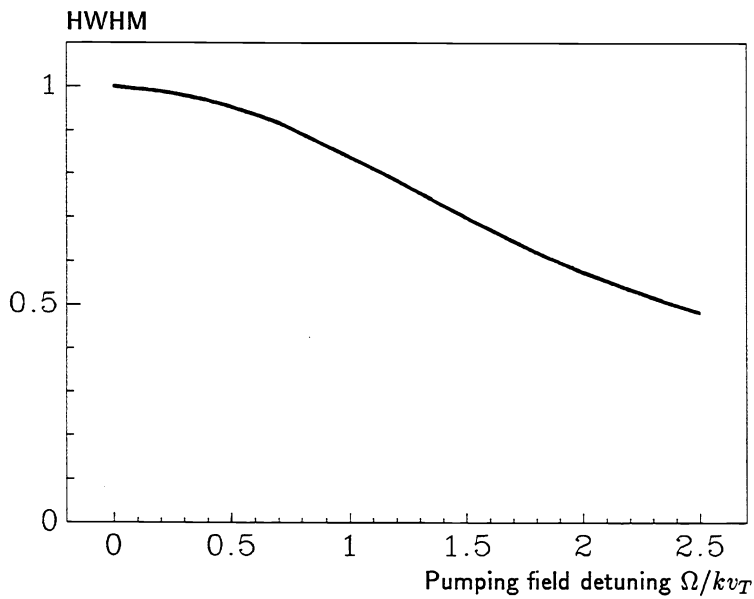


Fig.5. Nonlinear resonance within the first order of the perturbation theory in the field intensity. Different curves correspond to distinct values of detuning Ω (from left to right $\Omega/kv_T = 0; 0.5; 1; 1.5; 2; 2.5$). Parameters are assumed to be $\Gamma = 0.5 \cdot 10^{-2} kv_T$, $\Gamma_j = 10^{-3} kv_T$, $\nu = 10^{-5} kv_T$.



Half-width on half a maximum of nonlinear resonance as a function of detuning Ω/kv_T normalized to unit magnitude.

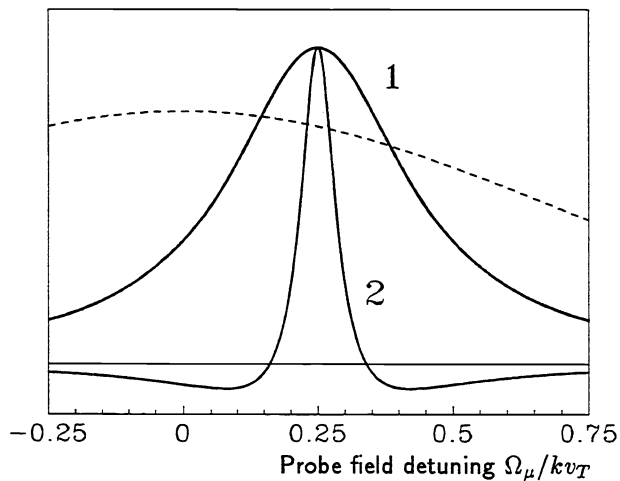


Fig.6. The profiles of nonlinear saturation (curve 1) and interference (curve 2) resonances as functions of Ω_μ . $\Gamma = 0.1kv_T$, $\Gamma_j = 0.04kv_T$, $\Omega = 0.25kv_T$. The dashed line is a Doppler contour.

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